SOLVING SYSTEMS OF NONLINEAR EQUATIONS USING INTERVAL ARITHMETIC AND TERM CONSISTENCY

ABSTRACT

One embodiment of the present invention provides a computer-based system for solving a system of nonlinear equations specified by a vector function, $\mathbf{f}$, wherein $\mathbf{f}(\mathbf{x}) = \mathbf{0}$ represents $f_1(\mathbf{x}) = 0, f_2(\mathbf{x}) = 0, f_3(\mathbf{x}) = 0, \ldots, f_n(\mathbf{x}) = 0$, wherein $\mathbf{x}$ is a vector $(x_1, x_2, x_3, \ldots x_n)$. The system operates by receiving a representation of an interval vector $\mathbf{X} = (X_1, X_2, \ldots, X_n)$, wherein for each dimension, i, the representation of X_i includes a first floating-point number, a_i, representing the left endpoint of X_i, and a second floating-point number, b_i, representing the right endpoint of X_i. For each nonlinear equation $f_i(\mathbf{x}) = 0$ in the system of equations $\mathbf{f}(\mathbf{x}) = \mathbf{0}$, each individual component function $f_i(\mathbf{x})$ can be written in the form $f_i(\mathbf{x}) = g(x'_j) - h(\mathbf{x})$ *or* $g(x'_j) = h(\mathbf{x})$, where g can be analytically inverted so that an explicit expression for x'_j can be obtained: $x'_j = g^{-1}(h(\mathbf{x}))$. Next, the system substitutes the interval vector element X_j into the modified equation to produce the equation $g(X'_j) = h(\mathbf{X})$, and solves for $X'_j = g^{-1}(h(\mathbf{X}))$. The system then intersects X'_j with X_j and replaces X_j in the interval vector $\mathbf{X}$ to produce a new interval vector $\mathbf{X}^+$, wherein the new interval vector $\mathbf{X}^+$ contains all solutions of the system of equations $\mathbf{f}(\mathbf{x}) = \mathbf{0}$ within the interval vector $\mathbf{X}$, and wherein the width of the new interval vector $\mathbf{X}^+$ is less than or equal to the width of the interval vector $\mathbf{X}$.